

RATIONAL NUMBERS 6TH GRADE

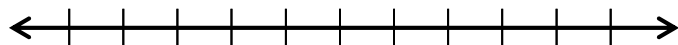
Lesson 1: Positive and Negative Numbers on the Number Line—Opposite Direction and Value

Classwork

Exploratory Challenge: Constructing a Number Line

Exercises

Complete the diagrams. Count by ones to label the number lines.



1. Plot your point on both number lines.
2. Show and explain how to find the opposite of your number on both number lines.
3. Mark the opposite on both number lines.
4. Choose a group representative to place the opposite number on the class number lines.
5. Which group had the opposite of the number on your index card?

Lesson 2: Real-World Positive and Negative Numbers and Zero

Classwork

Example 1: Take it to the Bank

For Tim's 13th birthday, he received \$150 in cash from his mom. His dad took him to the bank to open a savings account. Tim gave the cash to the banker to deposit into the account. The banker credited Tim's new account \$150 and gave Tim a receipt. One week later, Tim deposited another \$25 that he had earned as allowance. The next month, Tim asked his dad for permission to withdraw \$35 to buy a new video game. Tim's dad explained that the bank would charge \$5 for each withdrawal from the savings account and that each withdrawal and charge results in a debit to the account.

Read Example 1 silently. In the first column, write down any words and definitions you know. In the second column, write down any words you do not know.

Words I Already <u>Know</u> :	Words I <u>Want</u> to Know:	Words I <u>Learned</u> :

In the third column, write down any new words and definitions that you learn during the discussion.

Exercises 1–2

1. Read Example 1 again. With your partner, number the events in the story problem. Write the number above each sentence to show the order of the events.

For Tim's 13th birthday, he received \$150 in cash from his mom. His dad took him to the bank to open a savings account. Tim gave the cash to the banker to deposit into the account. The banker credited Tim's new account \$150 and gave Tim a receipt. One week later, Tim deposited another \$25 that he had earned as allowance. The next month, Tim asked his dad for permission to withdraw \$35 to buy a new video game. Tim's dad explained that the bank would charge \$5 for each withdrawal from the savings account and that each withdrawal and charge results in a debit to the account.

2. Write each individual description below as an integer. Model the integer on the number line using an appropriate scale.

EVENT	INTEGER	NUMBER LINE MODEL
Open a bank account with \$0.		
Make a \$150 deposit.		
Credit an account for \$150.		
Make a deposit of \$25.		
A bank charge of \$5.		
A withdrawal of \$35.		

Example 2: How Hot, How Cold?

Temperature is commonly measured using one of two scales, Celsius or Fahrenheit. In the United States the Fahrenheit system continues to be the accepted standard for non-scientific use. All other countries have adopted Celsius as the primary scale in use. The thermometer shows how both scales are related.

- a. The boiling point of water is 100°C . Where is 100 degrees Celsius located on the thermometer to the right?

- b. On a vertical number line, describe the position of the integer that represents 100°C .

- c. Write each temperature as an integer.

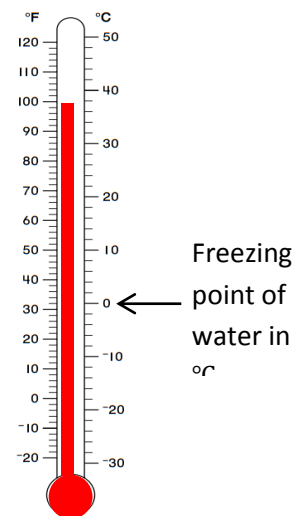
- i. The temperature shown to the right in $^{\circ}\text{F}$:

- ii. The temperature shown to the right in $^{\circ}\text{C}$:

- iii. Freezing point of water in Celsius:

- d. If someone tells you your body temperature is 98.6° , what scale are they using? How do you know?

- e. Does the temperature 0 degrees mean the same thing on both scales?



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Exercises 3–5

3. Write each word under the appropriate column, "Positive Number" or "Negative Number".

Gain	Loss	Deposit	Credit	Debit	Charge	Below Zero	Withdraw	Owe
				Receive				

Positive Number	Negative Number

4. Write an integer to represent each of the following situations:

- a. A company loses \$345,000 in 2011.

- b. You earned \$25 for dog sitting _____

- c. Jacob owes his dad \$5.

- d. The temperature at the sun's surface is about $5,600^{\circ}\text{C}$.

- e. The temperature outside is 4 degrees below zero.

- f. A football player lost 10 yards when he was tackled.

5. Describe a situation that can be modeled by the integer -15 . Explain what zero represents in the situation.



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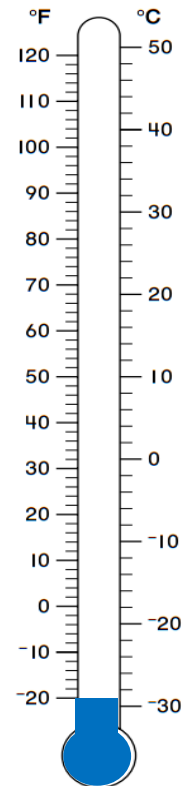
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6. Express each situation as an integer in the space provided.

- a. A gain of 56 points in a game. _____
- b. A fee charged of \$2.50. _____
- c. A temperature of 32 degrees below zero. _____
- d. A 56 yard loss. _____
- e. The freezing point of water in Celsius. _____
- f. A \$12,500 deposit. _____

For questions 7-10, use the thermometer to the right.

7. Each sentence is stated *incorrectly*. Rewrite the sentence to correctly describe each situation.
- a. The temperature is -10 degrees Fahrenheit below zero.
 - b. The temperature is -22 degrees Celsius below zero.
8. Mark the integer on the thermometer that corresponds to the temperature given.
- a. 70°F
 - b. 12°C
 - c. 110°F
 - d. -4°C
9. The boiling point of water is 212°F . Can this thermometer be used to record the temperature of a boiling pot of water? Explain.
10. Kaylon shaded the thermometer to represent a temperature of 20 degrees below zero Celsius as shown in the diagram. Is she correct? Why or why not? If necessary, describe how you would fix Kaylon's shading.

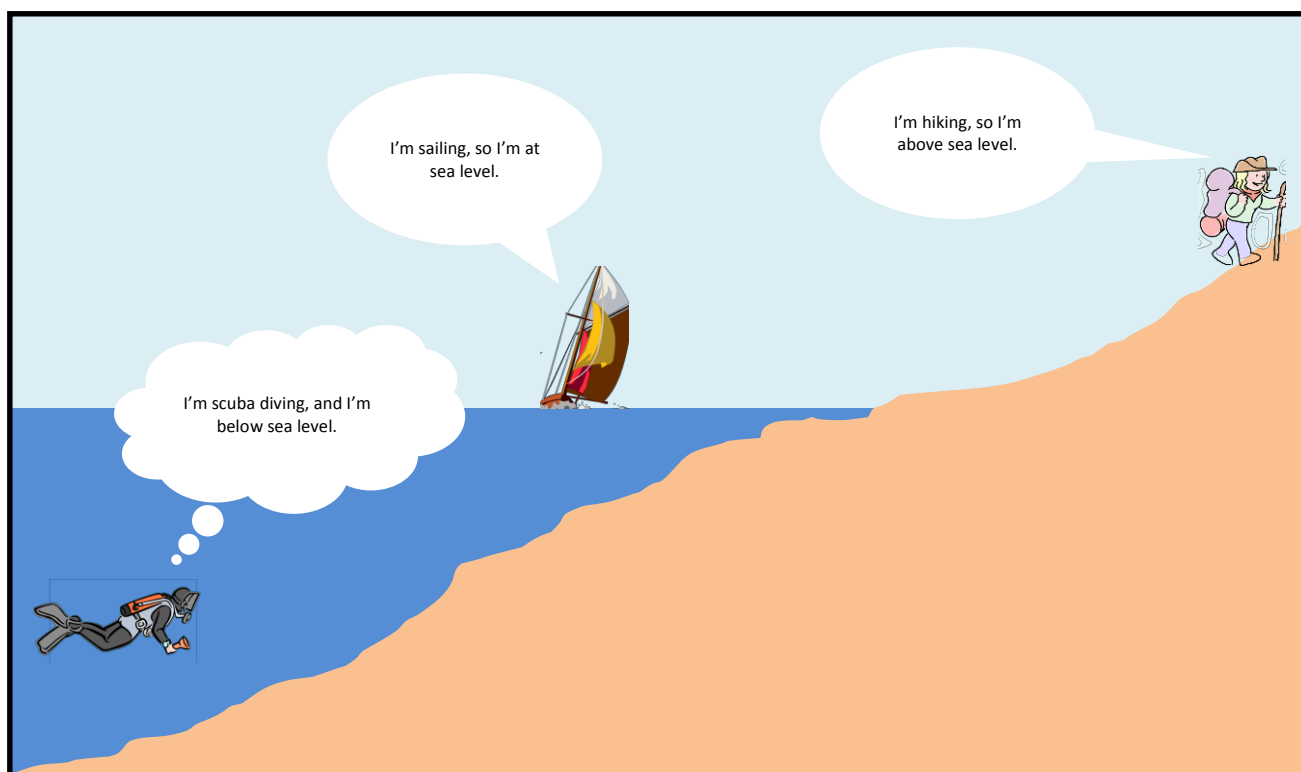


Lesson 3: Real-World Positive and Negative Numbers and Zero

Classwork

Example 1: A Look at Sea Level

The picture below shows three different people participating in activities at three different elevations. With a partner, discuss what you see. What do you think the word *elevation* means in this situation?



Exercises

Refer back to Example 1. Use the following information to answer Exercises 1 and 2.

- The diver is 30 feet below sea level.
- The sailor is at sea level.
- The hiker is 2 miles (10,560 feet) above sea level.

1. Write an integer to represent each situation.

2. Use an appropriate scale to graph each of the following situations on the number line to the right.



Also, write an integer to represent both situations.

a. A hiker is 15 feet above sea level.

b. A diver is 20 feet below sea level.

3. For each statement there are two related statements: *i* and *ii*. Determine which related statement is expressed correctly (*i* and *ii*), and circle it. Then correct the other related statement so that both parts, *i* and *ii*, are stated correctly.

a. A submarine is submerged 800 feet below sea level.

i. The depth of the submarine is -800 feet below sea level.

ii. 800 feet below sea level can be represented by the integer -800 .

b. The elevation of a coral reef with respect to sea level is given as -250 feet.

i. The coral reef is 250 feet below sea level.

ii. The depth of the coral reef is -250 feet below sea level.

Lesson 4: The Opposite of a Number

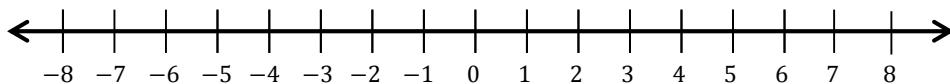
Classwork

Exercise 1: Walk the Number Line

- Each integer has an opposite, denoted $-a$; $-a$ and a are opposites if they are on opposite sides of zero and the same distance from zero on the number line.

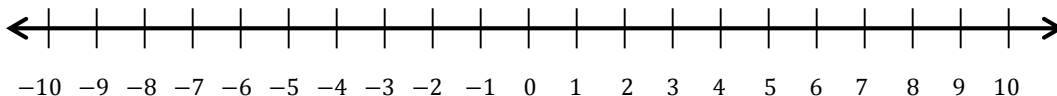
Example 1: Every Number has an Opposite

Locate the number 8 and its opposite on the number line. Explain how they are related to zero.



Exercises 2–3

- Locate the opposites of the numbers on the number line.
 - 9
 - 2
 - 4
 - 7

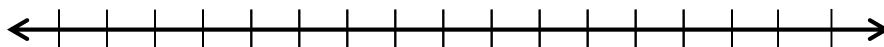


3. Write the integer that represents the opposite of each situation. Explain what zero means in each situation.
- 100 feet above sea level.
 - 32 degrees below zero.
 - A withdrawal of \$25.

Example 2: A Real World Example

Maria decides to take a walk along Central Avenue to purchase a book at the bookstore. On her way, she passes the Furry Friends Pet Shop and goes in to look for a new leash for her dog. The Furry Friends Pet Shop is seven blocks west of the bookstore. After she leaves the bookstore, she heads east for seven blocks and stops at Ray's Pet Shop to see if she can find a new leash at a better price. Which locations, if any, are the furthest from Maria while she is at the bookstore?

Determine an appropriate scale and model the situation on the number line below.

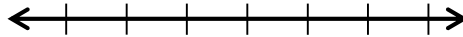


Explain your answer. What does zero represent in the situation?

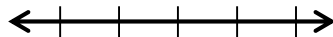
Exercises 4–8

Read each situation carefully and answer the questions.

4. On a number line, locate and label a credit of \$15 and a debit for the same amount from a bank account. What does zero represent in this situation?

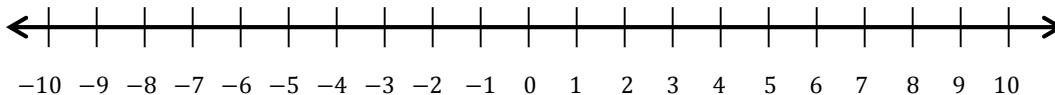


5. On a number line, locate and label 20°C below zero and 20°C above zero. What does zero represent in this situation?



6. Write the opposite of each number and label the points on the number line.

- a. Point A: The opposite of 9.
- b. Point B: The opposite of -4 .
- c. Point C: The opposite of -7 .
- d. Point D: The opposite of 0.
- e. Point E: The opposite of 2.



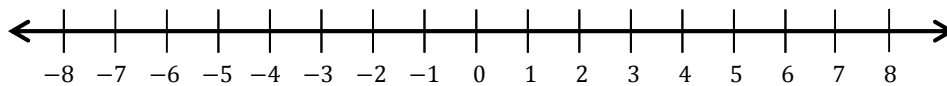
7. Study the first example. Write the integer that represents the opposite of each real-world situation. In words, write the meaning of the opposite.
- a. An atom's positive charge of 7.
 - b. A deposit of \$25.
 - c. 3,500 feet below sea level.
 - d. A rise of 45°C .
 - e. A loss of 13 pounds.

Lesson 5: The Opposite of a Number's Opposite

Classwork

Opening Exercise

1. Locate the number -2 and its opposite on the number line below.



2. Write an integer that represents each of the following:
- 90 feet below sea level
 - \$100 of debt
 - 2°C above zero
3. Joe is at the ice cream shop and his house is 10 blocks north of the shop. The park is 10 blocks south of the ice cream shop. When he is at the ice cream shop, is Joe closer to the park or his house? How could the number zero be used in this situation? Explain.

Example 1: The Opposite of an Opposite of a Number

What is the opposite of the opposite of 8? How can we illustrate this number on a number line?

- What number is 8 units to the right of 0? _____
- How can you illustrate locating the opposite of 8 on this number line? What is the opposite of 8? ____
- Use the same process to locate the opposite of -8 . What is the opposite of -8 ? _____



- The opposite of an opposite of a number is _____.

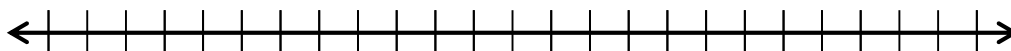
Exercise

Complete the table using the cards in your group.

Person	Card (a)	Opposite of Card ($-a$)	Opposite of Opposite of Card $-(a)$

- Write the opposite of the opposite of -10 as an equation.
- In general, the opposite of the opposite of a number is the _____.
- Provide a real-world example of this rule. Show your work.

7. Read each real-world description. Write the integer that represents the opposite of the opposite. Show your work to support your answer.
- A temperature rise of 15 degrees Fahrenheit.
 - A loss of 10 pounds.
8. Write the integer that represents the statement. Locate and label each point on the number line below.
- The opposite of a gain of 6.
 - The opposite of a deposit of \$10.
 - The opposite of the opposite of 0.
 - The opposite of the opposite of 4.
 - The opposite of the opposite of a loss of 5.



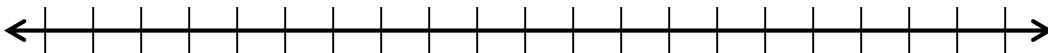
Lesson 6: Rational Numbers on the Number Line

Example 1: Graphing Rational Numbers

If b is a nonzero whole number, then the unit fraction $\frac{1}{b}$ is located on the number line by dividing the segment between 0 and 1 into b segments of equal length. One of the b segments has 0 as its left endpoint; the right endpoint of this segment corresponds to the unit fraction $\frac{1}{b}$.

The fraction $\frac{a}{b}$ is located on the number line by joining a segments of length $\frac{1}{b}$, so that: (1) the left endpoint of the first segment is 0, and (2) the right endpoint of each segment is the left endpoint of the next segment. The right endpoint of the last segment corresponds to the fraction $\frac{a}{b}$.

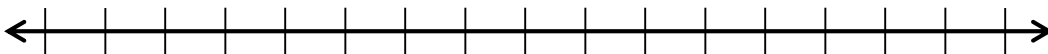
Locate and graph the number $\frac{3}{10}$ and its opposite on a number line.



Exercise 1

Use what you know about the points, $-\frac{7}{4}$ and its opposite, to graph both points on the number line below.

The fraction, $-\frac{7}{4}$, is located between which two consecutive integers? Explain your reasoning.

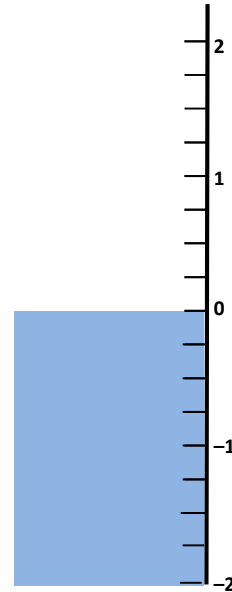


On the number line, each segment will have an equal length of _____. In the fraction $-\frac{7}{4}$, the numerator is _____ and the denominator is _____. The fraction is located between _____ and _____.

Example 2: Rational Numbers and the Real World

The water level of a lake rose 1.25 feet after it rained. Answer the questions below using the diagram below.

- a. Write a rational number to represent the situation.
- b. What two integers is 1.25 between on a number line?
- c. Write the length of each segment on the number line as a decimal and a fraction.
- d. What will be the water level after it rained? Graph the point on the number line.
- e. After two weeks of rain, the water level of the lake is the opposite of the water level before it rained. What will be the new water level? Graph the point on the number line. Explain how you got your answer.
- f. State a rational number that is not an integer whose value is less than 1.25, and describe its location between two consecutive integers on the number line.

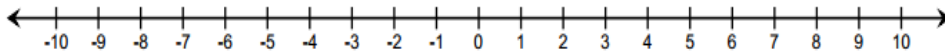
**Exercise 2****Our Story Problem**

Lesson 7: Ordering Integers and Other Rational Numbers

Classwork

Exercise 1

- 1.
- a. Graph 7 and its opposite on the number line. Graph 5 and its opposite on the number line.



- b. Where does 7 lie in relation to 5 on the number line?
- c. Where does the opposite of 7 lie on the number line in relation to the opposite of 5?
- d. I am thinking of 2 numbers. The first number lies to the right of the second number on a number line. What can you say about the location of their opposites? (If needed, refer to your number line diagram.)

Example 1

The record low temperatures for a town in Maine for January and February are -20 and -19 degrees Fahrenheit respectively. Order the numbers from least to greatest. Explain how you arrived at the order.

Exercises 2–4

For each problem, order the rational numbers from least to greatest. First read the problem, then draw a number line diagram, and finally, write/explain the answer. (Allow time for whole-group presentations.)

2. Jon's time for running the mile in gym class is 9.2 minutes. Jacky's time is 9.18 minutes. Who ran the mile in less time?

3. Mrs. Rodriguez is a teacher at Westbury Middle School. She gives bonus points on tests for outstanding written answers and deducts points for answers that are not written correctly. She uses rational numbers to represent the points. She wrote the following on the students' papers: Student A: -2 points, Student B: -2.5 points. Did Student A or Student B perform worse on the test?

4. A carp is swimming approximately $8\frac{1}{4}$ feet beneath the water's surface, and a sunfish is swimming approximately $3\frac{1}{2}$ feet beneath the water's surface. Which fish is swimming further beneath the water's surface?

Example 2

Henry, Janon, and Clark are playing a card game. The object of the game is to finish with the most points. The scores at the end of the game are: Henry: -7 , Janon: 0 , and Clark: -5 . Who won the game? Who came in last place? Use a number line model and explain how you arrived at your answer.

Exercises 5–6

For each problem, order the rational numbers from least to greatest by first reading the problem, then drawing a number line diagram, and finally, explaining your answer.

5. Henry, Janon, and Clark are playing another round of the card game. Their scores this time are as follows: Clark: -1 , Janon: -2 , and Henry: -4 . Who won? Who came in last place?

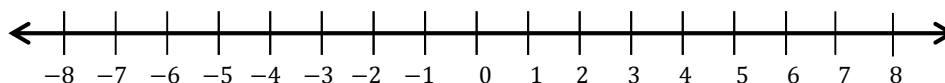
6. In the table below, list each set of rational numbers in order from least to greatest. Then list their opposites. Then list the opposites in order from least to greatest. The first example has been completed for you.

Rational Numbers	Ordered from Least to Greatest	Opposites	Opposites ordered from Least to Greatest
$-7.1, -7.25$	$-7.25, -7.1$	$7.25, 7.1$	$7.1, 7.25$
$\frac{1}{4}, -\frac{1}{2}$			
$2, -10$			
$0, 3\frac{1}{2}$			
$-5, -5.6$			
$24\frac{1}{2}, 24$			
$-99.9, -100$			
$-0.05, -0.5$			
$-0.7, 0$			
$100.02, 100.04$			

6. For each row, what pattern do you notice between the numbers in the 2nd and 4th columns? Why is this so?

Closing Exercise: What Is the Value of Each Number and Which Is Larger?

Use your teacher's verbal clues and this number line to determine which number is larger.



Lesson 8: Ordering Integers and Other Rational Numbers

Classwork

Exercise 1

- Students are each given four index cards or small slips of paper. Each student must independently choose four non-integer rational numbers, and write each one on a slip of paper. At least two of the numbers must be negative.
- Students order their rational numbers from least to greatest by sliding their slips of paper into the correct order. The teacher walks around the room to check for understanding and provide individual assistance. Students may use the number line in their student materials to help determine the order.
- Once all students have arranged their numbers into the correct order, they shuffle them and switch with another student.
- Students arrange the new set of cards they received into the correct order from least to greatest.
- The pairs of students who exchanged cards discuss their solutions and come to a consensus.

Example 1

Sam has \$10 in the bank. He owes his friend Hank \$2.25. He owes his sister \$1.75. Consider the three rational numbers related to this story of Sam's money. Write and order them from least to greatest.

Exercises 2–4

For each problem, list the rational numbers that relate to each situation. Then, order them from least to greatest; and explain how you made your determination.

- During their most recent visit to the optometrist (eye doctor), Kadijsha and her sister Beth had their vision tested. Kadijsha's vision in her left eye was -1.50 and her vision in her right eye was the opposite number. Beth's vision was -1.00 in her left eye and $+0.25$ in her right eye.
- There are three letters in Ms. Thomas's mailbox: a bill from the phone company for \$38.12, a bill from the electric company for \$67.55, and a tax refund check for \$25.89. (A bill is money that you owe someone and a tax refund check is money that you receive from someone.)

4. Monica, Jack and Destiny each had their arm length measured for an experiment in science class. They compared their arm lengths to a standard of 22 inches. The listing below shows in inches how each student's arm length compares to 22 inches. Order these rational numbers from least to greatest.

Monica: $-\frac{1}{8}$

Jack: $1\frac{3}{4}$

Destiny: $-\frac{1}{2}$

Example 2: Ordering Rational Numbers from Greatest to Least

Jason is entering college and has opened a checking account, which he will use for college expenses. His parents gave him \$200 to deposit into the account. Jason wrote a check for \$85.00 to pay for his Calculus book and a check for \$25.34 to pay for miscellaneous school supplies. Write the three rational numbers related to the balance in Jason's checking account in order from greatest to least.

Exercise 5–6

For each problem, list the rational numbers that relate to each situation in order from greatest to least. Lastly, explain how you arrived at their order.

5. The following are the current monthly bills that Mr. McGraw must pay:

\$122.00 Cable and Internet

\$73.45 Gas and Electric

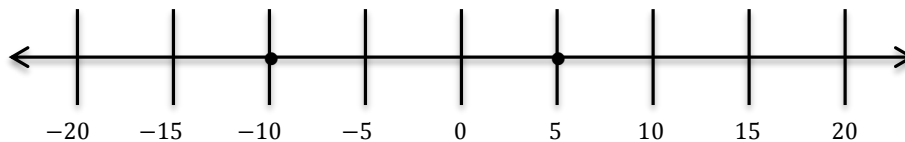
\$45.00 Cell phone

6. Arrange the following rational numbers in order from greatest to least: $-\frac{1}{3}$, 0 , $-\frac{1}{5}$, $\frac{1}{8}$.

Lesson 9: Comparing Integers and Other Rational Numbers

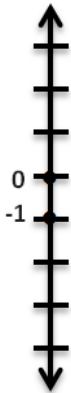
Classwork

Example 1: Interpreting Number Line Models to Compare Numbers



Exercises

1. Create a real-life situation that relates to the points shown in the number line model. In your write-up, be sure to describe the relationship between the values of the two points and how it relates to their order on the number line.



For each problem, determine if you *agree or disagree* with the representation. Then *defend your stance* by citing specific details in your writing.

2. Felicia needs to write a story problem that relates to the order in which the numbers $-6\frac{1}{2}$ and -10 are represented on a number line. She writes the following:

“During a recent football game, our team lost yards on two consecutive downs. We lost $6\frac{1}{2}$ yards on the first down. During the second down our quarterback was sacked for an additional 10 yard loss. On the number line, I represented this situation by first locating $-6\frac{1}{2}$. I located the point by moving $6\frac{1}{2}$ units to the left of zero. Then I graphed the second point by moving 10 units to the left of 0.”

3. Manuel looks at a number line diagram that has the points $-\frac{3}{4}$ and $-\frac{1}{2}$ graphed. He writes the following related story:

“I borrowed 50 cents from my friend, Lester. I borrowed 75 cents from my friend, Calvin. I owe Lester less than I owe Calvin.”

4. Henry located $2\frac{1}{4}$ and 2.1 on a number line. He wrote the following related story:

“In gym class both Jerry and I ran for 20 minutes. Jerry ran $2\frac{1}{4}$ miles and I ran 2.1 miles. I ran a farther distance.

5. Sam looked at two points that were graphed on a vertical number line. He saw the points -2 and 1.5 . He wrote the following description:

“I am looking at a vertical number line that shows the location of two specific points. The first point is a negative number, and so it is below zero. The second point is a positive number, and so it is above zero. The negative number is -2 . The positive number is $\frac{1}{2}$ unit more than the negative number.”

6. Claire draws a vertical number line diagram and graphs two points: -10 and 10 . She writes the following related story:
“These two locations represent different elevations. One location is 10 feet above sea level, and one location is 10 feet below sea level. On a number line, 10 feet above sea level is represented by graphing a point at 10, and 10 feet below sea level is represented by graphing a point at -10 .”
7. Mrs. Kimble, the sixth grade math teacher, asked the class to describe the relationship between two points on the number line, 7.45 and 7.5, and to create a real-world scenario. Jackson writes the following story:
“Two friends, Jackie and Jennie, each brought money to the fair. Jackie brought more than Jennie. Jackie brought \$7.45 and Jennie brought \$7.50. Since 7.45 has more digits than 7.5, it would come after 7.5 on the number line,
or to the right, so it is a greater value.”
8. Justine graphs the points associated with the following numbers on a vertical number line: $-1\frac{1}{4}$, $-1\frac{1}{2}$, and 1. He then writes the following real-world scenario:
“The nurse measured the height of three sixth grade students and compared their heights to the height of a typical sixth grader. Two of the students’ heights were below the typical height, and one was above the typical height. The point whose coordinate is 1 represented the student who had a height that was 1 inch above the typical height. Given this information, Justine determined that the student represented by the point associated with $-1\frac{1}{4}$ was the shortest of the three students.”

Lesson 10: Writing and Interpreting Inequality Statements Involving Rational Numbers

Classwork

Opening Exercises

“The amount of money I have in my pocket is less than \$5 but greater than \$4.”

- One possible value for the amount of money in my pocket is:
- Write an inequality statement comparing the possible value of the money in your pocket to \$4.
- Write an inequality statement comparing the possible value of the money in your pocket to \$5.

Exercises 1–4

- Graph your answer from the Opening Exercise, part (a) on the number line below.
- Also graph the points associated with 4 and 5 on the number line.
- Explain in words how the location of the three numbers on the number line supports the inequality statements you wrote in parts (b) and (c)
- Write one inequality statement that shows the relationship among all three numbers.



Example 1: Writing Inequality Statements Involving Rational Numbers

Write one inequality statement to show the relationship between the following shoe sizes: $10\frac{1}{2}$, 8, and 9.

a. From Least to Greatest:

b. From Greatest to Least:

Example 2: Interpreting Data and Writing Inequality Statements

Mary is comparing the rainfall totals for May, June and July. The data is reflected in the table below. Fill in the blanks below to create inequality statements that compare the *Changes in Total Rainfall* for each month (the right-most column of the table).

Month	This Year's Total Rainfall (in inches)	Last Year's Total Rainfall (in inches)	Change in Total Rainfall from last year to this year (in inches)
May	2.3	3.7	-1.4
June	3.8	3.5	0.3
July	3.7	3.2	0.5

Order the Changes in Total Rainfall _____
 From Least to Greatest From Greatest to Least

In this case, does the greatest number indicate the greatest change in rainfall? Explain.

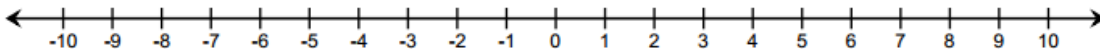
Exercises 5–8

5. Mark's favorite football team lost yards on two plays back-to-back. They lost **3** yards on the first play. They lost 1 yard on the second play. Write an inequality statement using integers to compare the forward progress made on each play.
6. Sierra had to pay the school for two textbooks that she lost. One textbook costs **\$55** and the other cost **\$75**. Her mother wrote two separate checks for each expense. Write two integers that represent the change to her mother's checking account balance. Then write an inequality statement that shows the relationship between these two numbers.
7. Jason ordered the numbers, **-70** , **-18** , and **-18.5** , from least to greatest by writing the following statement:
 $-18 < -18.5 < -70$. Is this a true statement? Explain.
8. Write a real-world situation that is represented by the following inequality: **$-19 < 40$** . Explain the position of the numbers on a number line.

Lesson 11: Absolute Value—Magnitude and Distance

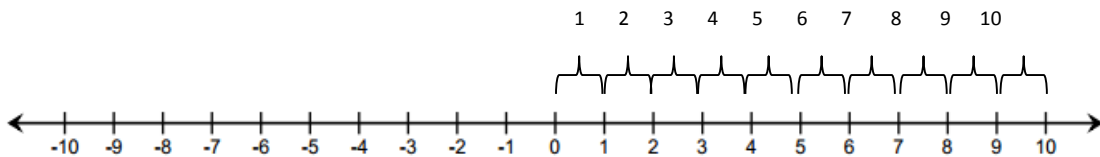
Classwork

Opening Exercise



Example 1: The Absolute Value of a Number

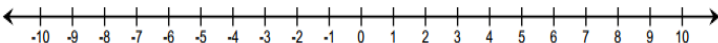
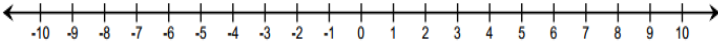
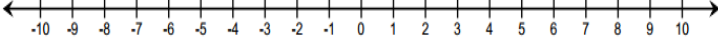
The absolute value of ten is written as: $|10|$. On the number line, count the number of units from 10 to 0. How many units is 10 from 0? $|10| =$



What other number has an absolute value of 10? Why?

The **absolute value** of a number is the distance between the number and zero on the number line.

Complete the following chart.

Number	Absolute Value	Number Line Diagram	Different Number with the Same Absolute Value
-6			
8			
-1			

Example 2: Using Absolute Value to Find Magnitude

Mrs. Owens received a call from her bank because she had a checkbook balance of -45 dollars. What was the magnitude of the amount overdrawn?

The **magnitude** of a quantity is found by taking the absolute value of its numerical part

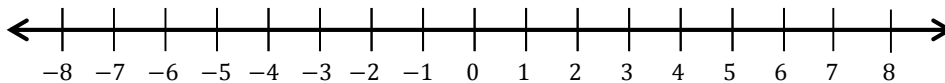
Exercises 1-16

For each scenario below, use absolute value to determine the magnitude of each quantity.

1. Maria was sick with the flu and her weight change as a result of it is represented by -4 pounds. How much weight did Maria lose?
2. Jeffrey owes his friend \$5. How much is Jeffrey's debt?
3. The elevation of Niagara Falls, which is located between Lake Erie and Lake Ontario, is **326** feet. How far is this above sea level?
4. How far below zero is -16 degrees Celsius?

5. Frank received a monthly statement for his college savings account. It listed a deposit of \$100 as +100.00. It listed a withdrawal of \$25 as -25.00 . The statement showed an overall ending balance of \$835.50. How much money did Frank add to his account that month? How much did he take out? What is the total amount Frank has saved for college?
6. Meg is playing a card game with her friend Iona. The cards have positive and negative numbers printed on them. Meg exclaims: "The absolute value of the number on my card equals 8!" What is the number on Meg's card?
7. List a positive and negative number whose absolute value is greater than 3. Explain how to justify your answer using the number line.
8. Which of the following situations can be represented by the absolute value of 10? Check all that apply.
- ☐ The temperature is 10 degrees below zero. Express this as an integer.
 - ☐ Determine the size of Harold's debt if he owes \$10.
 - ☐ Determine how far -10 is from zero on a number line.
 - ☐ 10 degrees is how many degrees above zero?
9. Julia used absolute value to find the distance between 0 and 6 on a number line. She then wrote a similar statement to represent the distance between 0 and -6 . Below is her work. Is it correct? Explain.
- $$|6| = 6 \text{ and } |-6| = -6$$
10. Use absolute value to represent the amount, in dollars, of a \$238.25 profit.
11. Judy lost 15 pounds. Use absolute value to represent the number of pounds Judy lost.

12. In math class, Carl and Angela are debating about integers and absolute value. Carl said two integers can have the same absolute value and Angela said one integer can have two absolute values. Who is right? Defend your answer.
13. Jamie told his math teacher: “Give me any absolute value, and I can tell you two numbers that have that absolute value.” Is Jamie correct? For any given absolute value, will there always be two numbers that have that absolute value?
14. Use a number line to show why a number and its opposite have the same absolute value.



15. A bank teller assisted two customers with transactions. One customer made a \$25.00 withdrawal from a savings account. The other customer made a \$15 deposit. Use absolute value to show the size of each transaction. Which transaction involved more money?
16. Which is farther from zero: $-7\frac{3}{4}$ or $7\frac{1}{2}$? Use absolute value to defend your answer.

Lesson 12: The Relationship Between Absolute Value and Order

Classwork

Opening Exercise

Record your integer values in order from least to greatest in the space below.

Example 1: Comparing Order of Integers to the Order of their Absolute Values

Write an inequality statement relating the ordered integers from the Opening Exercise. Below each integer write its absolute value.

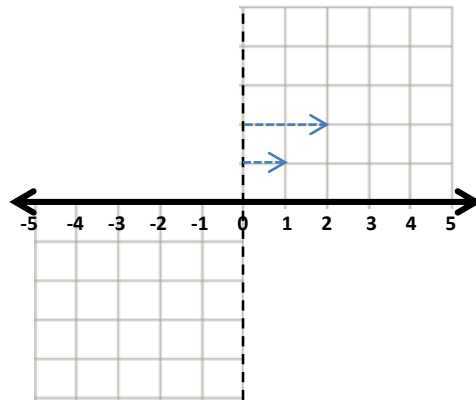
Rewrite the integers that are not circled in the space below. How do these integers differ from the ones you circled?

Rewrite the negative integers in ascending order and their absolute values in ascending order below them.

Describe how the order of the absolute values compares to the order of the negative integers.

Example 2: The Order of Negative Integers and their Absolute Values

Draw arrows starting at the dashed line (zero) to represent each of the integers shown on the number line below. The arrows that correspond with 1 and 2 have been modeled for you.



As you approach zero from the left on the number line, the integers _____, but the absolute values of those integers _____. This means that the order of negative integers is _____ the order of their absolute values.

Exercise 1

Complete the steps below to order these numbers:

$$\left\{ 2.1, -4\frac{1}{2}, -6, 0.25, -1.5, 0, \quad 3.9, -6.3, -4, 2\frac{3}{4}, \quad 3.99, -9\frac{1}{4} \right\}$$

- Separate the set of numbers into positive and negative values and zero in the top cells below.
- Write the absolute values of the rational numbers (order does not matter) in the bottom cells below.

<div style="border: 2px solid orange; border-radius: 50%; padding: 20px; margin-bottom: 10px;">Negative Rational Numbers</div> <div style="border: 2px solid orange; border-radius: 50%; padding: 20px;">Absolute Values</div>	Zero 0	<div style="border: 2px solid orange; border-radius: 50%; padding: 20px; margin-bottom: 10px;">Positive Rational Numbers</div> <div style="border: 2px solid orange; border-radius: 50%; padding: 20px;">Absolute Values</div>
--	---------------	--

- c. Order each subset of absolute values.

0

- d. Order each subset of rational numbers.

0

- e. Order the whole given set of rational numbers.

Exercise 2

- a. Find a set of four integers such that their order and that of the order of their absolute values is the same.
- b. Find a set of four integers such that their order and the order of their absolute values are opposite.
- c. Find a set of four non-integer rational numbers such that their order and the order of their absolute values is the same.
- d. Find a set of four non-integer rational numbers such that their order and the order of their absolute values are opposite.
- e. Order all of your numbers from parts (a)–(d) in the space below. This means you should be ordering 16 numbers from least to greatest.

Lesson 13: Statements of Order in the Real World

Classwork

Opening Exercise

A radio disc jockey reports that the temperature outside his studio has changed 10 degrees since he came on the air this morning. Discuss with your group what listeners can conclude from this report.

Example 1: Ordering Numbers in the Real World

A \$25 credit and a \$25 charge appear similar, yet they are very different.

Describe what is similar about the two transactions.

How do the two transactions differ?

Exercises 1–4

- Scientists are studying temperatures and weather patterns in the Northern Hemisphere. They recorded temperatures (in degrees Celsius) in the table below, as reported in emails from various participants. Represent each reported temperature using a rational number. Order the rational numbers from least to greatest. Explain why the rational numbers that you chose appropriately represent the given temperatures.

Temperatures as Reported	8 below zero	12	−4	13 below zero	0	2 above zero	6 below zero	−5
Temperature (°C)								

2. Jami's bank account statement shows the transactions below. Represent each transaction as a rational number describing how it changes Jami's account balance, then order the rational numbers from greatest to least. Explain why the rational numbers that you chose appropriately reflect the given transactions.

Listed Transactions	Debit \$12.20	Credit \$4.08	Charge \$1.50	Withdrawal \$20.00	Deposit \$5.50	Debit \$3.95	Charge \$3.00
Change to Jami's Account							

3. During the summer, Madison monitors the water level in her parents' swimming pool to make sure it is not too far above or below normal. The table below shows the numbers she recorded in July and August. Represent each measurement as a rational number to describing how the water levels compare to normal, then order the rational numbers from least to greatest. Explain why the rational numbers that you chose appropriately reflect the given water levels.

Madison's Readings	$\frac{1}{2}$ inch above normal	$\frac{1}{4}$ inch above normal	$\frac{1}{2}$ inch below normal	$\frac{1}{8}$ inch above normal	$1\frac{1}{4}$ inch below normal	$\frac{3}{8}$ inch below normal	$\frac{3}{4}$ inch below normal
Compared to Normal							

4. Changes in the weather can be predicted by changes in the barometric pressure. Over several weeks, Stephanie recorded changes in barometric pressure seen on her barometer to compare to local weather forecasts. Her observations are recorded in the table below. Use rational numbers to record the indicated changes in the pressure in the second row of the table. Order the rational numbers from least to greatest.

Barometric Pressure Change	Rise 0.04	Fall 0.21	Rise 0.2	Fall 0.03	Rise 0.1	Fall 0.09	Fall 0.14
Barometric Pressure Change							

Example 2: Using Absolute Value to Solve Real-World Problems

The captain of a fishing vessel is standing on the deck at 23 feet above sea level. He holds a rope tied to his fishing net that is below him underwater at a depth of 38 feet.

Draw a diagram using a number line, then use absolute value to compare the lengths of rope in and out of the water.

Example 3: Making Sense of Absolute Value and Statements of Inequality

A recent television commercial asked viewers “Do you have over \$10,000 in credit card debt?”

What types of numbers are associated with the word “debt” and why? Write a number that represents the value from the television commercial.

Give one example of “over \$10,000 in credit card debt” then write a rational number that represents your example.

How do the debts compare and how do the rational numbers that describe them compare? Explain.

Lesson 14: Ordered Pairs

Classwork

Example 1: The *Order* in Ordered Pairs

The first number of an ordered pair is called the _____.

The second number of an ordered pair is called the _____.

Example 2: Using Ordered Pairs to Name Locations

Describe how the ordered pair is being used in your scenario. Indicate what defines the first coordinate and what defines the second coordinate in your scenario.

Scenario 1: The seats in a college football stadium are arranged into 210 sections, with 144 seats in each section. Your ticket to the game indicates the location of your seat using the ordered pair of numbers (123, 37). Describe the meaning of each number in the ordered pair and how you would use them to find your seat.

Scenario 2: Airline pilots use measurements of longitude and latitude to determine their location and to find airports around the world. Longitude is measured as $0-180^\circ$ east or $0-180^\circ$ west of a line stretching from the North Pole to the South Pole through Greenwich, England called the prime meridian. Latitude is measured as $0-90^\circ$ north or $0-90^\circ$ south of the Earth's Equator. A pilot has the ordered pair (90° west, 30° north). What does each number in the ordered pair describe? How would the pilot locate the airport on a map? Would there be any confusion if a pilot were given the ordered pair (90° , 30°)? Explain.

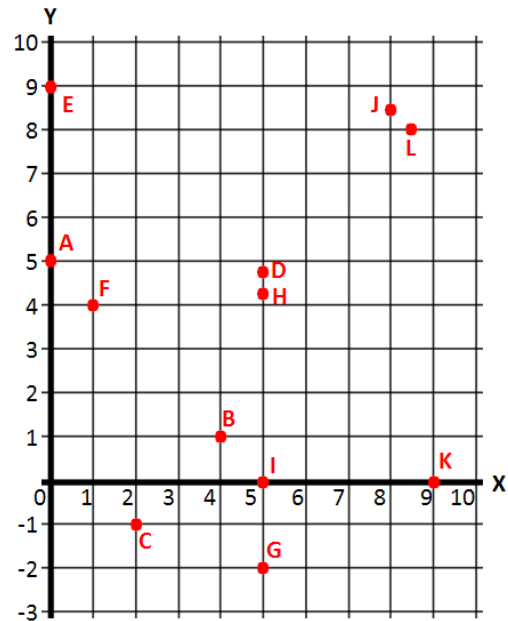
Scenario 3: Each room in a school building is named by an ordered pair of numbers that indicates the number of the floor on which the room lies, followed by the sequential number of the room on the floor from the main staircase. A new student at the school is trying to get to science class in room 4–13. Describe to the student what each number means and how she should use the number to find her classroom. Suppose there are classrooms below the main floor. How might these rooms be described?

Exercises

The first coordinates of the ordered pairs represent the numbers on the line labeled x , and the second coordinates represent the numbers on the line labeled y .

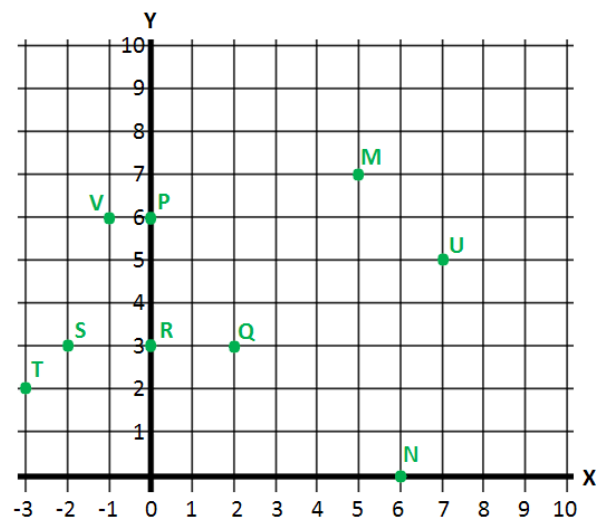
1. Name the letter from the grid (on the right) that corresponds with each ordered pair of numbers below.

- | | |
|--------------|---------------|
| a. $(1, 4)$ | e. $(0, 5)$ |
| b. $(4, 1)$ | f. $(8.5, 8)$ |
| c. $(5, -2)$ | g. $(5, 4.2)$ |
| d. $(2, -1)$ | h. $(0, 9)$ |



2. List the ordered pair of numbers that corresponds with each letter from the grid below.

- | | |
|--------------|--------------|
| a. Point M | f. Point S |
| b. Point N | g. Point T |
| c. Point P | h. Point U |
| d. Point Q | i. Point V |
| e. Point R | |



Lesson 15: Locating Ordered Pairs on the Coordinate Plane

Classwork

Example 1: Extending the Axes Beyond Zero

The point below represents zero on the number line. Draw a number line to the right starting at zero. Then, follow directions as provided by the teacher.



Example 2: Components of the Coordinate Plane

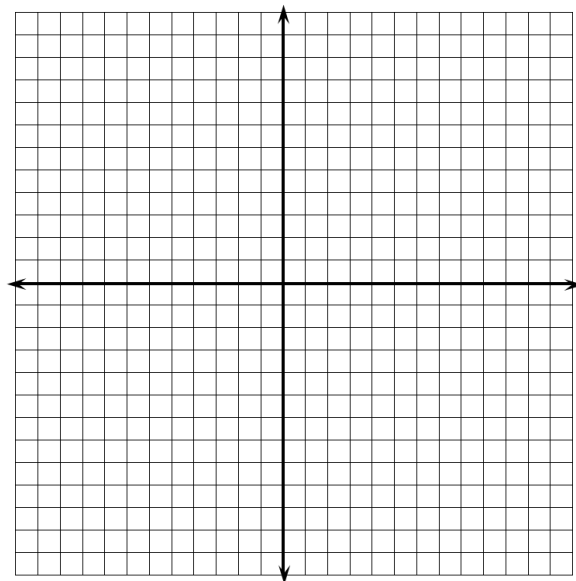
All points on the coordinate plane are described with reference to the origin. What is the origin, and what are its coordinates?

To describe locations of points in the coordinate plane we use _____ of numbers. Order is important, so on the coordinate plane we use the form (_____). The first coordinate represents the point's location from zero on the _____-axis, and the second coordinate represents the point's location from zero on the _____-axis.

Exercises

1. Use the coordinate plane below to answer parts (a)–(c):

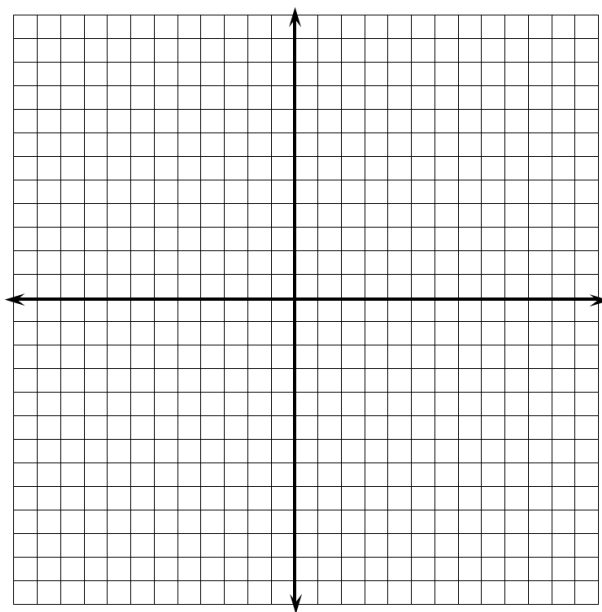
- Graph at least five points on the x -axis and label their coordinates.
- What do the coordinates of your points have in common?
- What must be true about any point that lies on the x -axis? Explain.



2. Use the coordinate plane to answer parts (a)–(c):

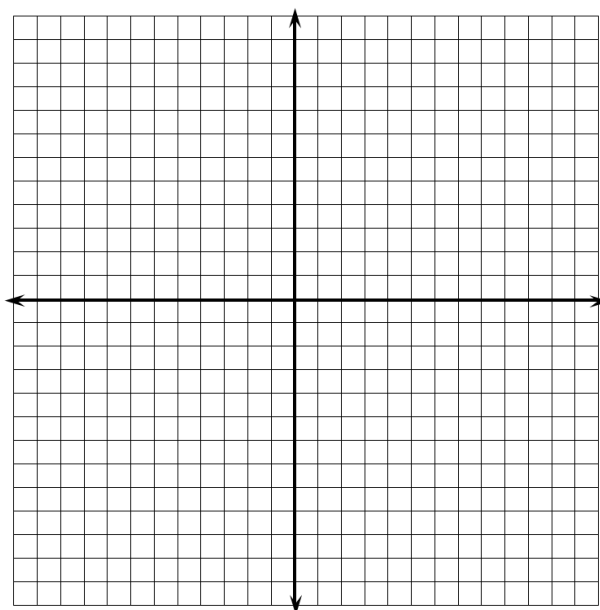
- Graph at least five points on the y -axis and label their coordinates.
- What do the coordinates of your points have in common?
- What must be true about any point that lies on the y -axis? Explain.

3. If the origin is the only point with 0 for both coordinates, what must be true about the origin?

Example 3: Quadrants of the Coordinate Plane**Exercises**

4. Locate and label each point described by the ordered pairs below. Indicate which of the quadrants the points lie in.

- A. $(7, 2)$
- B. $(3, -4)$
- C. $(1, -5)$
- D. $(-3, 8)$
- E. $(-2, -1)$



5. Write the coordinates of at least one other point in each of the four quadrants.
- a. Quadrant I

 - b. Quadrant II

 - c. Quadrant III

 - d. Quadrant IV
6. Do you see any similarities in the points within each quadrant? Explain your reasoning.

Lesson 16: Symmetry in the Coordinate Plane

Classwork

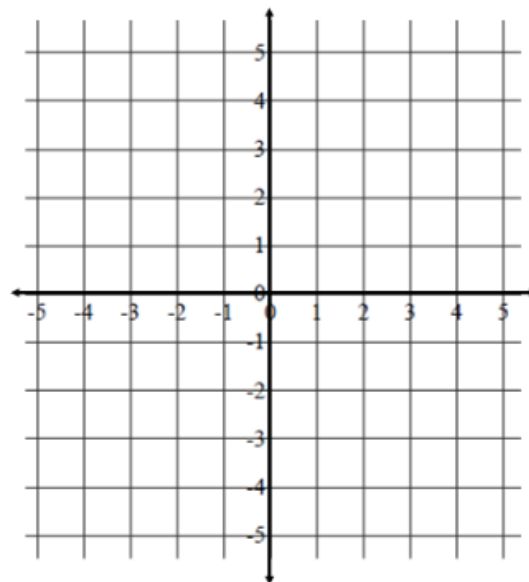
Opening Exercise

Give an example of two opposite numbers and describe where the numbers lie on the number line. How are opposite numbers similar and how are they different?

Example 1: Extending Opposite Numbers to the Coordinate Plane

Extending Opposite Numbers to the Coordinates of Points on the Coordinate Plane

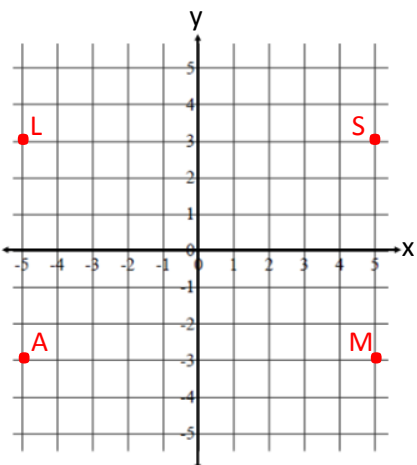
Locate and label your points on the coordinate plane to the right. For each given pair of points in the table below, record your observations and conjectures in the appropriate cell. Pay attention to the absolute values of the coordinates and where the points lie in reference to each axis.



	$(3, 4)$ and $(-3, 4)$	$(3, 4)$ and $(3, -4)$	$(3, 4)$ and $(-3, -4)$
Similarities of Coordinates			
Differences of Coordinates			
Similarities in Location			
Differences in Location			
Relationship between Coordinates and Location on the Plane			

Exercise

In each column, write the coordinates of the points that are related to the given point by the criteria listed in the first column of the table. Point $S(5,3)$ has been reflected over the x - and y -axes for you as a guide and its images are shown on the coordinate plane. Use the coordinate grid to help you locate each point and its corresponding coordinates.

Given Point:	$S(5,3)$	$(-2,4)$	$(3,-2)$	$(-1,-5)$	
Reflected across the x -axis.					
Reflected across the y -axis.					
Reflected first across the x -axis then across the y -axis.					
Reflected first across the y -axis then across the x -axis.					

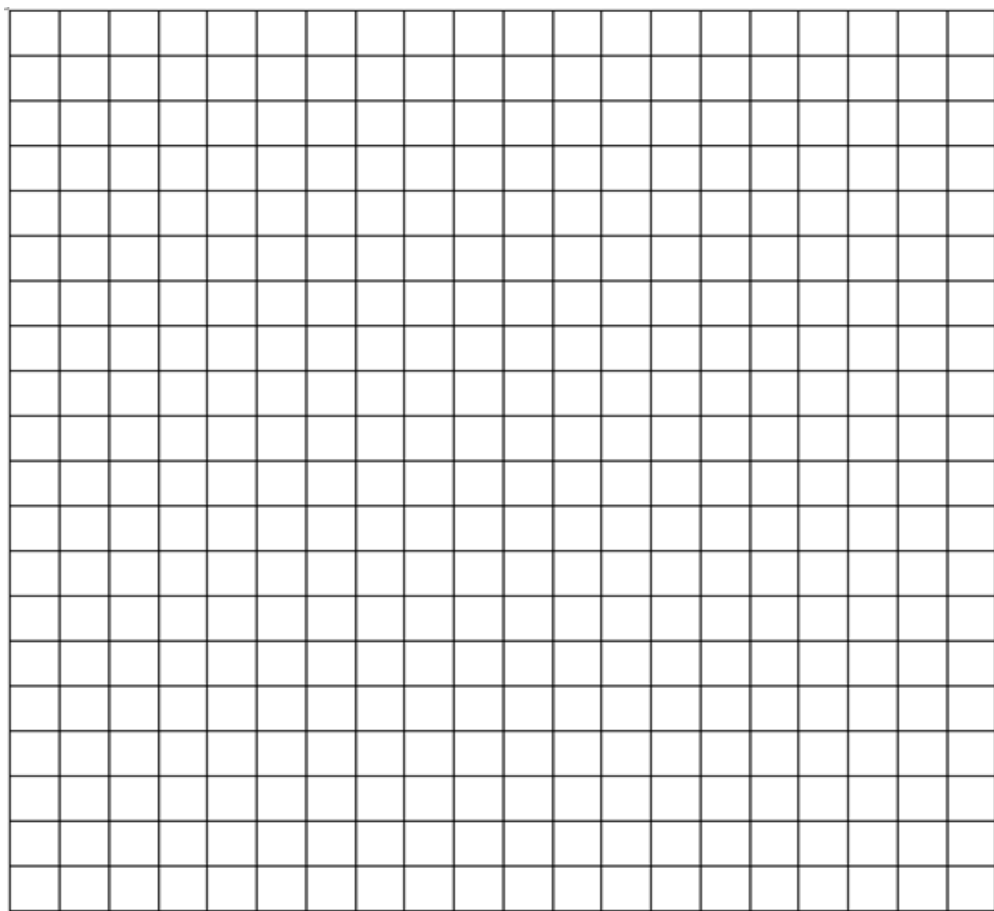
- When the coordinates of two points are (x, y) and $(-x, y)$, what line of symmetry do the points share? Explain.
- When the coordinates of two points are (x, y) and $(x, -y)$, what line of symmetry do the points share? Explain.

Lesson 17: Drawing the Coordinate Plane and Points on the Plane

Classwork

Opening Exercise

Draw all necessary components of the coordinate plane on the blank 20×20 grid provided below, placing the origin at the center of the grid and letting each grid line represent 1 unit.



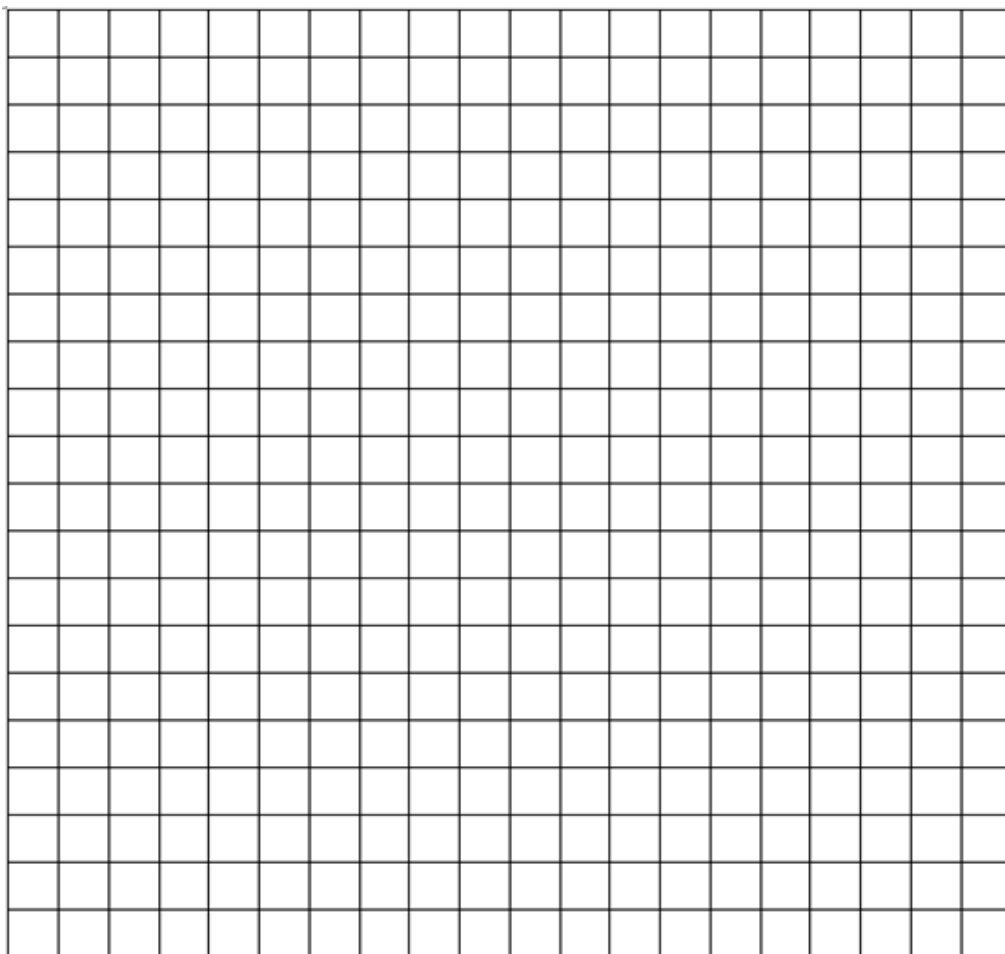
Example 1: Drawing the Coordinate Plane using a 1:1 Scale

Locate and label the points $\{(3,2), (8,4), (-3,8), (-2,-9), (0,6), (-1,-2), (10,-2)\}$ on the grid above.

Example 2: Drawing the Coordinate Plane Using an Increased Number Scale

Draw a coordinate plane on the grid below, then locate and label the following points:

$$\{(-14, 20), (-23, 35), (10, -35), (16, 10), (27, -40)\}$$



Lesson 18: Distance on the Coordinate Plane

Classwork

Opening Exercise

Four friends are touring on motorcycles. They come to an intersection of two roads; the road they are on continues straight, and the other is perpendicular to it. The sign at the intersection shows the distances to several towns. Draw a map/diagram of the roads and use it and the information on the sign to answer the following questions:

Albertsville ← 8 mi
Blossville ↑ 3 mi
Cheyenne ↑ 12 mi
Dewey Falls → 6 mi

What is the distance between Albertsville and Dewey Falls?

What is the distance between Blossville and Cheyenne?

On the coordinate plane, what represents the intersection of the two roads?

Example 1: The Distance Between Points on an Axis

What is the distance between $(-4,0)$ and $(5,0)$?

What do the ordered pairs have in common and what does that mean about their location in the coordinate plane?

How did we find the distance between two numbers on the number line?

Use the same method to find the distance between $(-4,0)$ and $(5,0)$.

Example 2: The Length of a Line Segment on an Axis

What is the length of the line segment with endpoints $(0, -6)$ and $(0, -11)$?

What do the ordered pairs of the endpoints have in common and what does that mean about the line segment's location in the coordinate plane?

Find the length of the line segment described by finding the distance between its endpoints $(0, -6)$ and $(0, -11)$.

Example 3: Length of a Horizontal or Vertical Line Segment that does Not Lie on an Axis

Find the length of the line segment with endpoints $(-3, 3)$ and $(-3, -5)$.

What do the endpoints, which are represented by the ordered pairs, have in common? What does that tell us about the location of the line segment on the coordinate plane?

Find the length of the line segment by finding the distance between its endpoints.

Exercises

1. Find the lengths of the line segments whose endpoints are given below. Explain how you determined that the line segments are horizontal or vertical.

a. $(-3, 4)$ and $(-3, 9)$

b. $(2, -2)$ and $(-8, -2)$

c. $(-6, -6)$ and $(-6, 1)$

d. $(-9, 4)$ and $(-4, 4)$

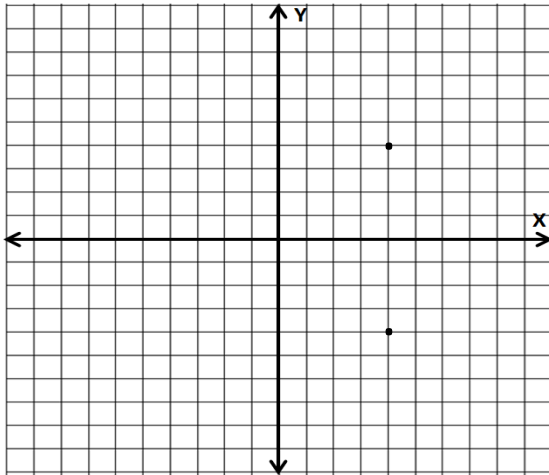
e. $(0, -11)$ and $(0, 8)$

Lesson 19: Problem-Solving and the Coordinate Plane

Classwork

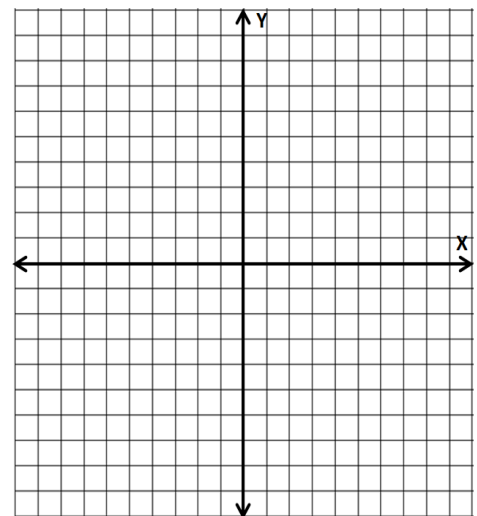
Opening Exercise

In the coordinate plane, find the distance between the points using absolute value.



Exercises

1. Locate and label $(4, 5)$ and $(4, -3)$. Draw the line segment between the endpoints given on the coordinate plane. How long is the line segment that you drew? Explain.
2. Draw a horizontal line segment starting at $(4, -3)$ that has a length of 9 units. What are the possible coordinates of the other endpoint of the line segment? (There is more than one answer.)



Which point do you choose to be the other endpoint of the horizontal line segment? Explain how and why you chose that point. Locate and label the point on the coordinate grid.

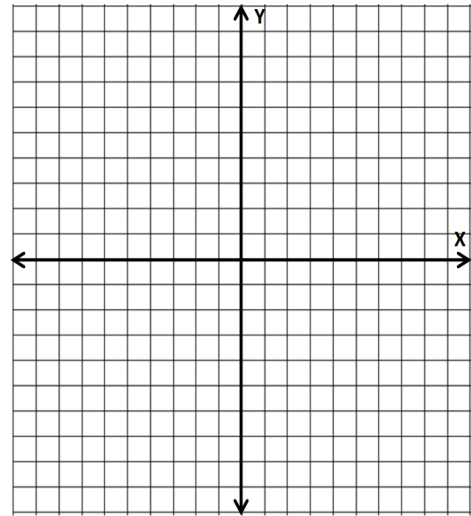
3. The two line segments that you have just drawn could be seen as two sides of a rectangle. Given this, the endpoints of the two line segments would be three of the vertices of this rectangle.
 - a. Find the coordinates of the fourth vertex of the rectangle. Explain how you find the coordinates of the fourth vertex using absolute value.
 - b. How does the fourth vertex that you found relate to each of the consecutive vertices in either direction? Explain.
 - c. Draw the remaining sides of the rectangle.
4. Using the vertices that you have found and the lengths of the line segments between them, find the perimeter of the rectangle.
5. Find the area of the rectangle.
6. Draw a diagonal line segment through the rectangle with opposite vertices for endpoints. What geometric figures are formed by this line segment? What are the areas of each of these figures? Explain.

EXTENSION [If time allows]: Line the edge of a piece of paper up to the diagonal in the rectangle. Mark the length of the diagonal on the edge of the paper. Align your marks horizontally or vertically on the grid and estimate the length of the diagonal to the nearest integer. Use that estimation to now estimate the perimeter of the triangles.

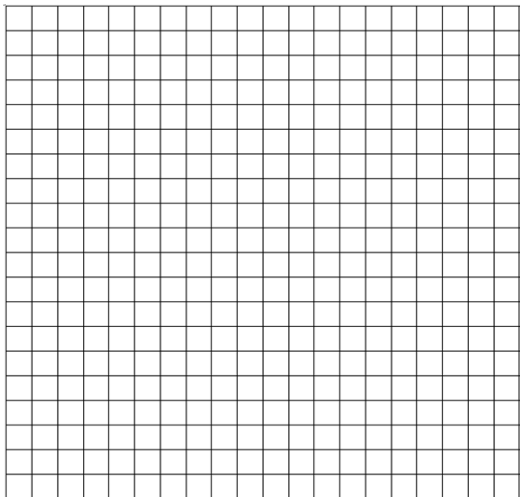
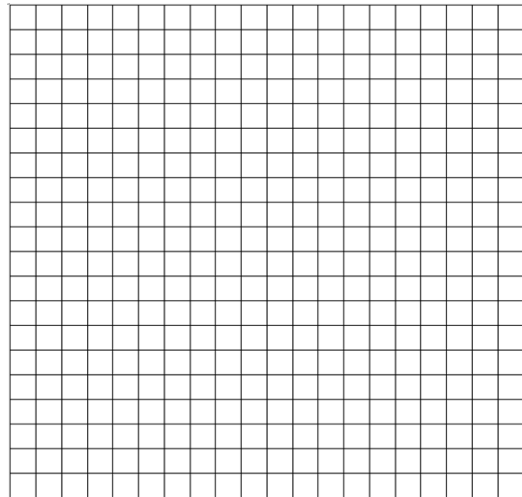
7. Construct a rectangle on the coordinate plane that satisfies each of the criteria listed below. Identify the coordinate of each of its vertices.

- Each of the vertices lies in a different quadrant.
- Its sides are either vertical or horizontal.
- The perimeter of the rectangle is 28 units.

Using absolute value, show how the lengths of the sides of your rectangle provide a perimeter of 28 units.



8. One endpoint of a line segment is $(-3, -6)$. The length of the line segment is 7 units. Find four points that could serve as the other endpoint of the given line segment.



9. Two of the vertices of a rectangle are $(1, -6)$ and $(-8, -6)$. If the rectangle has a perimeter of 26 units, what are the coordinates of its other two vertices?

10. A rectangle has a perimeter of 28 units, an area of 48 square units, and sides that are either horizontal or vertical. If one vertex is the point $(-5, -7)$ and the origin is in the interior of the rectangle, find the vertex of the rectangle that is opposite $(-5, -7)$.

